Introduction to AI

Lecture 12
Propositional Logic

Dr. Tamal Ghosh
Department of CSE
Adamas University

Knowledge Representation

Human brain has memory cells which store information as KNOWLEDGE and as per the knowledge various actions are performed in the real world. **In AI machines do replicate that using knowledge representation**.

•Knowledge representation is the part of AI where agents can think and thinking contributes to intelligent behavior.

•Information about the real world are stored in knowledge base which helps a computer understand and utilize the knowledge to solve the complex real world problems such as diagnosis a medical condition or communicating with humans in natural language.

•Knowledge representation is not just storing data into some database, but it also enables an intelligent machine to learn from that knowledge and experiences so that it can behave intelligently.

What Knowledge Represents?

Following are ‘represented’ in AI systems:

- **Object:** All the facts about objects in our world domain. E.g., Guitars contains strings, trumpets are brass instruments.
- **Events:** Events are the actions which occur in our world.
- **Performance:** It describe behavior which involves knowledge about how to do things.
- **Meta-knowledge:** It is knowledge about what we know or beyond.
- **Facts:** Facts are the truths about the real world and what we represent.
- **Knowledge-Base:** The central component of the knowledge-based agents is the knowledge base. It is represented as KB. The Knowledgebase is a group of the Sentences (Here, sentences are used as a technical term and not identical with the English language).

Knowledge: Knowledge is awareness or familiarity gained by experiences of facts, data, and situations. Following are the types of knowledge in artificial intelligence:

Ways to represent knowledge

- **Propositional Logic -** Propositional logic, also known as sentential logic, is that branch of logic that studies ways of combining or altering statements or propositions to form more complicated statements or propositions.

- **First Order Logic / Predicate Logic -** Predicate logic, first-order logic or quantified logic is a formal language in which propositions are expressed in terms of predicates, variables and quantifiers. It is an extension / different from propositional logic which lacks quantifiers.

Propositional Logic

Propositional logic (PL) is the simplest form of logic where all the statements are made by propositions. A proposition is a declarative statement which is either true or false. It is a technique of knowledge representation in logical and mathematical form.

Example:

1.a) It is Sunday.

2.b) The Sun rises from West (False proposition)

3.c) 3+3= 7(False proposition)

4.d) 5 is a prime number.

Basic Facts of PL

- Propositional logic is also called Boolean logic as it works on 0 and 1.
- In propositional logic, we use symbolic variables to represent the logic, and we can use any symbol for a representing a proposition, such A, B, C, P, Q, R, etc.
- Propositions can be either true or false, but it cannot be both.
- Propositional logic consists of an object, relations or function, and **logical connectives**.
- These connectives are also called logical operators.
- The propositions and connectives are the basic elements of the propositional logic.
- Connectives can be said as a logical operator which connects two sentences.
- A proposition formula which is always true is called **tautology**, and it is also called a valid sentence.
- A proposition formula which is always false is called **Contradiction**.
- A proposition formula which has both true and false values is called **Contingent proposition**.
- Statements which are questions, commands, or opinions are not propositions such as "**Where is Rohini**", "**How are you**", "**What is your name**", are not propositions.

Syntax of Propositions

The syntax of propositional logic defines the allowable sentences for the knowledge representation. There are two types of Propositions:

(1) Atomic Propositions (2) Compound propositions

•**Atomic Proposition:** Atomic propositions are the simple propositions. It consists of a single proposition symbol. These are the sentences which must be either true or false.

Example:

1.a) 2+2 is 4, it is an atomic proposition as it is a **true** fact.

2.b) "The Sun is cold" is also a proposition as it is a **false** fact.

•**Compound proposition:** Compound propositions are constructed by combining simpler or atomic propositions, using parenthesis and logical connectives.

Example:

1.a) "It is raining today, and street is wet."

2.b) "Ankit is a doctor, and his clinic is in Mumbai."

Logical Connectives

Logical connectives are used to connect two simpler propositions or representing a sentence logically. We can create compound propositions with the help of logical connectives. There are mainly five connectives, which are given as follows:

1.Negation: A sentence such as ¬ P is called negation of P. A literal can be either Positive literal or negative literal.

2.Conjunction: A sentence which has **Λ** connective such as, **P Λ Q** is called a conjunction.

Example: Rohan is intelligent and hardworking. It can be written as,

P= Rohan is intelligent,

Q= Rohan is hardworking. → PΛ Q.

Logical Connectives

3. **Disjunction:** A sentence which has V connective, such as **P V Q**. is called disjunction, where P and Q are the propositions.
   Example: "Ritika is a doctor or Engineer",
   Here P= Ritika is Doctor. Q= Ritika is Doctor, so we can write it as **P V Q**.

4.Implication: A sentence such as P → Q, is called an implication. Implications are also known as if-then rules. It can be represented as

If it is raining, then the street is wet.

Let P= It is raining, and Q= Street is wet, so it is represented as P → Q

5.Biconditional: A sentence such as **P⇔ Q is a Biconditional sentence, example If I am breathing, then I am alive**

P= I am breathing, Q= I am alive, it can be represented as P ⇔ Q.

Logical Connectives and Truth Tables

Connective symbols	Word	Technical term	Example
∧	AND	Conjunction	A ∧ B
∨	OR	Disjunction	A ∨ B
→	Implies	Implication	A → B
⇔	If and only if	Biconditional	A⇔ B
¬ or ~	Not	Negation	¬ A or ¬ B

For Negation:

P	¬ P
True	**False**
False	**True**

For Conjunction:

P	Q	P∧ Q
True	True	**True**
True	False	**False**
False	True	**False**
False	False	**False**

Truth table of three propositions

P	Q	R	¬R	P∨ Q	P∨Q→¬R
True	True	True	False	True	False
True	True	False	True	True	True
True	False	True	False	True	False
True	False	False	True	True	True
False	True	True	False	True	False
False	True	False	True	True	True
False	False	True	False	False	True
False	False	False	True	False	True

For disjunction:

P	Q	P ∨ Q.
True	True	**True**
False	True	**True**
True	False	**True**
False	False	**False**

For Implication:

P	Q	P→ Q
True	True	**True**
True	False	**False**
False	True	**True**
False	False	**True**

Precedence of Connectives

Precedence	Operators
First Precedence	Parenthesis
Second Precedence	Negation
Third Precedence	Conjunction(AND)
Fourth Precedence	Disjunction(OR)
Fifth Precedence	Implication
Six Precedence	Biconditional

Logical Equivalence

Logical equivalence is one of the features of propositional logic. Two propositions are said to be logically equivalent if and only if the columns in the truth table are identical to each other.

Let's take two propositions A and B, so for logical equivalence, we can write it as A⇔B. In below truth table we can see that column for ¬AV B and A→B, are identical hence A is Equivalent to B

A	B	¬A	¬AV B	A→B
T	T	F	**T**	**T**
T	F	F	**F**	**F**
F	T	T	**T**	**T**
F	F	T	**T**	**T**

Properties of Operators

- **Commutativity:**
  - P∧ Q= Q ∧ P, or
  - P ∨ Q = Q ∨ P.
- **Associativity:**
  - (P ∧ Q) ∧ R= P ∧ (Q ∧ R),
  - (P ∨ Q) ∨ R= P ∨ (Q ∨ R)
- **Identity element:**
  - P ∧ True = P,
  - P ∨ True= True.
- **Distributive:**
  - P∧ (Q ∨ R) = (P ∧ Q) ∨ (P ∧ R).
  - P ∨ (Q ∧ R) = (P ∨ Q) ∧ (P ∨ R).
- **DE Morgan's Law:**
  - ¬ (P ∧ Q) = (¬P) ∨ (¬Q)
  - ¬ (P ∨ Q) = (¬ P) ∧ (¬Q).
- **Double-negation elimination:**
  - ¬ (¬P) = P.

Limitations of Propositional logic

•We cannot represent relations like ALL, some, or none with propositional logic.

Example:

- **All the girls are intelligent.**
- **Some apples are sweet.**

•Propositional logic has limited expressive power.

•In propositional logic, we cannot describe statements in terms of their properties or logical relationships.